Application of complex-scaling method for few-body scattering

Rimantas Lazauskas*
IPHC, IN2P3-CNRS/Université Louis Pasteur BP 28, F-67037 Strasbourg Cedex 2, France

Jaume Carbonell†
LPSC, 53 Av. des Martyrs, 38026 Grenoble, France
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Formalism based on complex-scaling method is developed for solving the few particle scattering problem by employing only trivial boundary conditions. Several applications are presented proving efficiency of the method in describing elastic and three-body break-up reactions for Hamiltonians which may include both short and long-range interaction.

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I. INTRODUCTION

The quantum-mechanical problem of interacting particles is of the fundamental importance in theoretical physics, opening broad field of application related with the description of the microworld. Quick development of the computational techniques, following the rapid evolution of the computational power, provoked sizeable advance in multi-particle bound state problem: rigorous and accurate description of the systems formed of up to several or even dozens of particles have been obtained [1–4]. The progress in multi-particle scattering problem is moderated however. The major obstacle turns to be the rich variety of the reactions one should consider simultaneously and the resultant complexity of the wave function asymptotic structure. Till now only three-body system has been treated in a full extent, comprising elastic and break-up channels [5–7], whereas rigorous description by the same methods of the four particle scattering remains limited to the elastic and rearrangement channels [8–10]. Recently very courageous effort has been undertaken to apply Green Function Monte Carlo [11] and No-Core Shell model [12] calculations for the nucleon scattering on A≥4 nuclei, nevertheless these promising techniques remains limited to the description of the binary scattering process. Therefore a method which enable the scattering problem to be solved without explicit use of the asymptotic form of the wave function is of great importance.

The complex scaling method has been proposed [13, 14] and successfully applied for the resonance scattering [15], as has been demonstrated recently this method can be extended also for the scattering problem [16, 17]. In this study we propose novel method to solve quantum few-particle scattering problem based on complex scaling method, which allows to use trivial boundary conditions. We demonstrate success of this method in both calculating elastic and three particle break-up observables.

II. FORMALISM: 2-BODY CASE

1. *Short range interaction*

The complex scaling method has been proposed a while ago to treat the scattering problem for the exponentially bound potentials [13]. Idea is quite simple and can be summarized as follows. First one recast the Schrödinger equation into inhomogeneous (driven) form by splitting systems wave function into the sum $\Psi = \Psi^{sc} + \Psi^{in}$ containing the incident (free) $\Psi^{in}(\mathbf{r}) = \exp(i\mathbf{k}\cdot\mathbf{r})$ and the scattered $\Psi^{sc}(\mathbf{r})$ waves as:

$$(E - \widehat{H_0} - V(\mathbf{r}))\Psi^{sc}(\mathbf{r}) = V(\mathbf{r})\Psi^{in}(\mathbf{r}). \tag{1}$$

The scattered wave in the asymptote is represented by the outgoing wave $\Psi^{sc} \infty \exp(ikr)/r$. If one scale all the particle coordinates by a constant complex factor, i.e. $\overline{r}_i = e^{i\theta} r_i$ with $Im(e^{i\theta}) > 0$, the scattered wave vanish exponentially as $\overline{\Psi}^{sc} \infty \exp(-kr\sin\theta)$ with increasing particle separation r. Moreover ff the interaction is of short range (exponentially

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*Electronic address: rimantas.lazauskas@iphc.in2p3.fr
†Electronic address: jaume.carbonellm@lpsc.in2p3.fr

bound), the right hand side of eq. (1) also turns to zero for large r, enabling to solve former equation in a similar way as a bound state problem: using compact basis or by solving differential equation on a finite domain and by imposing $\overline{\Psi}^{sc}$ to vanish on its borders.

In practice one solves 2-body problem by expanding Schrödinger equation into partial waves:

$$(\frac{\hbar^2}{2\mu}k^2 - \widehat{H}_{0l}(r) - V_l(r))\psi_l^{sc}(r) = V_l(r)\psi_l^{in}(r), \tag{2}$$

The radial part of the incoming wave is represented by the regular Bessel functions $\psi_l^{in}(r) = j_l(kr)kr$ and with kinetic energy term given by

$$\widehat{H}_{0l}(r) = \frac{\hbar^2}{2\mu}\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}\right]. \tag{3}$$

after the complex scaling, this equation becomes:

$$(\frac{\hbar^2}{2\mu}k^2 - \widehat{H}_{0l}(re^{i\theta}) - V_l(re^{i\theta}))\overline{\psi}_l^{sc}(r) = V_l(re^{i\theta})\overline{\psi}_l^{in}(r), \tag{4}$$

The complex scaled inhomogeneous term is easily obtained by using analytical expressions for the regular Bessel function $\overline{\psi}_l^{in}(r) = j_l(kre^{i\theta})kre^{i\theta}$. Extraction of the scattering phase-shift may be done either directly by determining asymptotic normalization coefficient of the outgoing wave:

$$\overline{\psi}_l^{sc}(r) = A_l(r)\exp(ikre^{i\theta} - l\pi/2) \tag{5}$$

with scattering amplitude given by:

$$kf_l = e^{i\delta_l}\sin\delta_l = A_l(r \to \infty) \tag{6}$$

Other well known alternative is to use integral representation, which one gets after applying Green's theorem:

$$f_l = -\frac{2\mu}{\hbar^2}\int j_l(kre^{i\theta})V_l(re^{i\theta})(\overline{\psi}_l^{sc}(r) + \overline{\psi}_l^{in}(r))re^{2i\theta}dr \tag{7}$$

2. *Coulomb plus short range interaction*

If interaction contains long range term the problem turns to be quite different. The right hand side of eq. (1) after the complex scaling diverges and the $\Psi^{sc}(re^{i\theta})$ term is not anymore exponentially bound. In ref. [16, 17] exterior complex scaling was proposed as a solution to circumvent the problem due to diverging term on the right hand side of eq. (1).

In this paper, when considering problem of the interaction containing short range ($V^s(r)$) plus Coulomb term ($V^C(r) = \frac{\hbar^2\eta}{\mu r}$) we propose to stick with the standard smooth scaling procedure, however to employ analytically continued Coulomb waves to circumvent the problem of the divergence. In this case driven partial wave Schrödinger equation is written as:

$$(\frac{\hbar^2}{2\mu}k^2 - H_{0l}(r) - V^s(r) - V^C(r))\psi_l^{sc,C}(r) = V^s(r)\psi_l^{in,C}(r), \tag{8}$$

where $\psi_l^{in,C} = F_l(\eta, kr)$ is a regular Coulomb function, which is solution of the former Hamiltonian containing Coulomb interaction only. Asymptotically the scattered wave behaves as $\psi^{sc,C}\infty\exp(ikr - \eta\ln 2kr)$, and therefore vanish exponentially after the complex scaling.

$$\overline{\psi}_{sc}^C(r) = A_l(r)\exp(ikre^{i\theta} - \eta\ln 2kre^{i\theta} - l\pi/2). \tag{9}$$

Equation (8) may be readily solved with the vanishing boundary condition for $\overline{\psi}_{sc}^C(r)$, one simply must be able to continue analytically the regular Coulomb functions standing on the right hand side.

The scattering amplitude and Coulomb corrected phase shift due to short range interaction δ_l can be determined as previously from the asymptotic normalization coefficient

$$e^{-i\sigma_l} f_l = e^{i(\delta_l+\sigma_l)} \sin\delta_l = A_l(r\to\infty), \tag{10}$$

where σ_l is so-called Coulomb phase shift.

Alternatively, Green's theorem may be used to obtain integral relation similar to eq.(7):

$$f_l = -\frac{2\mu}{\hbar^2} e^{2i\sigma_l} \int F_l(\eta, kre^{i\theta}) V^s(re^{i\theta}) \overline{\Psi}^C(r) re^{2i\theta} dr \tag{11}$$

III. FORMALISM: 3-BODY CASE

3. Short range interaction

In sake of simplicity let consider a system of three identical spinless particles submitted to short range pairwise interactions. Only two vector variables are needed in the barycentric system, which may be one of the Jacobi pairs $\mathbf{x}_i = \mathbf{r}_j - \mathbf{r}_k$ and $\mathbf{y}_i = \frac{2}{\sqrt{3}}(\mathbf{r}_i - (\mathbf{r}_k + \mathbf{r}_j))$. The pair potential is assumed to support any number of two-particle bound states $\phi_m(\mathbf{x}_i)$ with eigenvalues ϵ_m and the angular momentum of this state l_m. The corresponding continuum state has relative momenta q_m, satisfying energy conservation relation $E = \frac{\hbar^2}{m} q_m^2 + \epsilon_m = K^2$; the second equality defines three-particle break-up momenta K.

Three particle problem we formulate by using Faddeev equations [18] in configuration space and by readily separating incoming wave of the particle scattered on a bound pair in the state $\phi_m(\mathbf{x}_i)$. Three-identical particle problem maybe concluded in a single equation:

$$(E - H_0 - V_i(\mathbf{x}_i))\psi^{sc}_{i,m}(\mathbf{x}_i,\mathbf{y}_i) - V_i(\mathbf{x}_i)\sum_{j\neq i}\psi^{sc}_{j,m}(\mathbf{x}_j,\mathbf{y}_j) = V_i(\mathbf{x}_i)\sum_{j\neq i}\phi_m(\mathbf{x}_j)\exp(i\mathbf{q}_m\cdot\mathbf{y}_j), \tag{12}$$

where $\psi^{sc}_{i,m}$ is the scattered part of the Faddeev amplitude, corresponding to the incoming particle i; while by V_i we denote a pair interaction of the particles j and k. Decomposition of the systems wave function into three Faddeev amplitudes permits to separate two-cluster particle channels, whereas three-body break-up component remains shared by the three Faddeev amplitudes. In the $y_i \to \infty$ asymptote the scattered part of the Faddeev amplitude i takes form

$$\psi^{sc}_{i,m}(\mathbf{x}_i,\mathbf{y}_i) \underset{y_i\to\infty}{=} A_m(\widehat{x}_i, \widehat{y}_i, x_i/y_i)\frac{\exp(iK\rho)}{\rho^{5/2}} + \sum_n f_{nm}(\widehat{y}_i)\phi_n(\mathbf{x}_i)\frac{\exp(iq_n y_i)}{|y_i|} \tag{13}$$

here $\rho = \sqrt{x_i^2 + y_i^2}$ is hyperradius of the system; $A_m(\widehat{x}_i, \widehat{y}_i, x_i/y_i)$ is a three-particle break-up amplitude; $f_{mn}(\widehat{y}_i)$ is two-body transition amplitude from channel m to channel n. In this expression sum runs over all open binary channels n.

One may easily see that the scattered part of the Faddeev amplitude might vanish for large hyperradius if particle coordinates are properly complex scaled and thus: $\overline{\mathbf{x}}_i = \mathbf{x}_i e^{i\theta}, \overline{\mathbf{y}}_i = \mathbf{y}_i e^{i\theta}$ and $\rho = \rho e^{i\theta}$. However in order to perform resolution of the problem on a finite grid one should ensure that the inhomogeneous term, standing in the right hand side of eq. (12), also vanishes outside the resolution domain. The inhomogeneous term is null, damped by the potential term, if x_i is large and thus falls outside of the interaction region. Alternatively for $x_i \ll y_i$, the modulus of the transformed Jacobi coordinates approach $x_j \approx \frac{\sqrt{3}}{2} y_i$; $y_j \approx y_i/2$ and

$$\phi_n(\mathbf{x}_j e^{i\theta})\exp(i\mathbf{q}_n\cdot\mathbf{y}_j e^{i\theta}) \propto \phi_n(\mathbf{x}_j e^{i\theta})\frac{\exp(-iq_n y_j e^{i\theta})}{|y_j|\, e^{i\theta}} \propto \exp(-k_n\frac{\sqrt{3}}{2}y_i\cos\theta)\frac{\exp(q_n\frac{y_i}{2}\sin\theta)}{y_i}, \tag{14}$$

here we explored the fact that the bound state wave function decrease exponentially in the asymptote with momenta $k_n = \sqrt{m\,|\epsilon_n|}/\hbar$. The last expression vanish for large y_i only if condition

$$\tan\theta < \frac{k_n\sqrt{3}}{q_n} = \sqrt{\frac{3\,|\epsilon_n|}{E + |\epsilon_n|}} \tag{15}$$

is satisfied [25]. Therefore if the scattering energy is large enough one is obliged to restrict the complex scaling angle to very small values

Extraction of the scattering observables maybe realized as previously in two different ways. Straightforward way is to extract transition amplitudes from the $y_i \to \infty$ asymptote of the solution $\overline{\psi}_i^{sc}(\mathbf{x}_i, \mathbf{y}_i)$, exploiting the fact that different scattering channels are mutually orthogonal:

$$f_{nm}(\widehat{y}_i) = C_n^{-1} |y_i| \exp(-iq_n y_i e^{i\theta}) \int \phi_n^*(\mathbf{x}_i e^{-i\theta}) \overline{\psi}_{i,m}^{sc}(\mathbf{x}_i, \mathbf{y}_i) e^{3i\theta} d^3\mathbf{x}_i, \tag{16}$$

where C_n is normalization coefficient of two-body wave function $\int \phi_n^*(\mathbf{x}_i e^{-i\theta}) \phi_n(\mathbf{x}_i e^{i\theta}) e^{3i\theta} d^3\mathbf{x}_i = C_n$. Break-up amplitude might be extracted from the $\overline{\psi}_i^{sc}(\mathbf{x}_i, \mathbf{y}_i)$ once all the two-body transition amplitudes are calculated relaying on eq. (16).

Alternatively one can employ Green's theorem. In this case integral relations might be obtained both for break-up and two-body transition amplitudes. For the transition amplitude one has:

$$f_{nm}(\widehat{y}_i) = -C_n^{-1} \frac{m}{\hbar^2} \int\int \phi_n^*(\mathbf{x}_i e^{-i\theta}) \frac{\exp(-iq_n y_i e^{i\theta})}{|y_i|} (V_j(\mathbf{x}_j e^{i\theta}) + V_k(\mathbf{x}_k e^{i\theta})) \overline{\Psi}_m(\mathbf{x}_i, \mathbf{y}_i) e^{6i\theta} d^3\mathbf{x}_i d^3\mathbf{y}_i \tag{17}$$

These integrals are convergent on the finite domain, if the following condition is satisfied:

$$\tan\theta < \frac{\sqrt{3}k_m}{q_m + 2q_n} = \frac{\sqrt{3\,|\epsilon_m|}}{\sqrt{|\epsilon_m| + E} + 2\sqrt{|\epsilon_n| + E}}, \tag{18}$$

which is stronger than condition eq. (15) [26].

For the break-up amplitude several different relations can be obtained [19], it seems that the one employing 2-body outgoing states $\overline{\phi}^{(+)}(p, \mathbf{x}_i)$, generated by the correspondingly scaled strong potential at relative momenta p, seems the most reliable numerically:

$$A_{i,m}(\widehat{x}_i, \widehat{y}_i, x_i/y_i) = \frac{m}{\hbar^2} \int\int \overline{\phi}^{(+)}(Kx_i/y_i, \mathbf{x}_i) \frac{\exp(-iq_n y_i e^{i\theta})}{|y_i|} V_i(\mathbf{x}_i e^{i\theta}) (\overline{\psi}_{j,m} + \overline{\psi}_{k,m}) e^{6i\theta} d^3\mathbf{x}_i d^3\mathbf{y}_i \tag{19}$$

In practice, calculations are performed by expanding former equations into partial waves. This pure technical issue is not subject of this paper and we refer interested reader to [19] for the details on the partial wave Faddeev equations. One should note that partial wave expansion has no effect on the validity of the presented method.

4. *Short range interaction plus Coulomb*

The former discussion can be readily extended for the case when particles interact via short range plus Coulomb forces. In this case we prefer to use Faddeev-Merkuriev equations [20], which elaborate Faddeev formalism by accommodating Coulomb force. Indeed, Faddeev equations, suppose free asymptotic behavior of the particles, in case of long range interaction become ill behaved due to noncompactness of their kernel. These equations can still provide satisfactory solution for bound state problem, but are impractical for the scattering case [27]. Faddeev equations does not shed light on the asymtotic behavior of the separate amplitudes when long range interaction is present. Idea of Merkuriev [20] is to split the Coulomb potential V^C into two parts (short and long range), $V^C = V^{s.C} + V^{l.C}$, by means of some arbitrary cut-off function χ:

$$V_i^{s.C}(x_i, y_i) = V_i^C(x_i)\chi_i(x_i, y_i) \qquad V_i^{l.C}(x_i, y_i) = V_i^C(x_i)[1 - \chi_i(x_i, y_i)] \tag{20}$$

and reshuffle long range terms. One is then left with a system of equivalent equations:

$$(E - H_0 - W_i - V_i^s)\psi_i^{sc} - V_i^s \sum_{i \neq j} (\psi_j^{sc} + \psi_{j,m}^{in}) = (W_i - V_i^l - V_i^{C.res})\psi_{i,m}^{in} \qquad W_i = V_i^l + V_j^l + V_k^l \tag{21}$$

by V_i^s we consider sum of the short range interaction $V_i(x_i)$ of the pair (jk) plus short range part of the Coulomb force $V_i^s = V_i^{s.C} + V_i$. The incoming state is defined by:

$$\psi_{i,m}^{in}(\mathbf{x}_i, \mathbf{y}_i) = \phi_m(\mathbf{x}_i)\varphi^C(q_m, \mathbf{y}_i) \tag{22}$$

where $\varphi^C(q_m, \mathbf{y}_i)$ is a plane wave of the incident particle i moderated by its residual Coulomb interaction with a cluster of particles (jk): $V_i^{C.res}(y_i) = \frac{4\hbar^2\eta_{i,jk}}{\sqrt{3}my_i}$. The $\phi_m(\mathbf{x}_i)$ is the eigenfunction of the $m-$th bound state of the particle pair (jk).

Former equations are solved as well as the scattering observables extracted in a similar way as for the Coulomb free case. For example transition amplitude, via Green's theorem, is expressed as:

$$f_{nm}^{C}(\widehat{y}_i) = -C_n^{-1}\frac{m}{\hbar^2}\int\int \phi_n^*(\mathbf{x}_i e^{-i\theta})\varphi^{C*}(q_m,\mathbf{y}_i e^{-i\theta}) \times$$
$$(V_j^s(\mathbf{x}_j e^{i\theta}) + V_k^s(\mathbf{x}_k e^{i\theta}) + W_i(\mathbf{x}_i e^{i\theta},\mathbf{y}_i e^{i\theta}) - V^{C.res}(y_i e^{i\theta}) - V_i(x_i e^{i\theta}))\overline{\Psi}_m(\mathbf{x}_i,\mathbf{y}_i)e^{6i\theta}d^3\mathbf{x}_i d^3\mathbf{y}_{,i} \quad (23)$$

with $V_i(x_i)$ representing full interaction between particle-pair (jk).

There is however formal difficulty associated with the extraction of the break-up amplitude for the case when all three particles are charged, since the asymptotic form of the break-up wave function is not known. One may still rely on the approximate relation employing Peterkop integral [21] as is claimed in [22].

IV. RESULTS

To test our approach we consider model of nucleons with mass $\frac{\hbar^2}{m} = 41.47\ MeV \cdot fm^2$, where strong part of nucleon-nucleon (NN) interaction is described by S-wave MT I-III potential, defined as:

$$V_S(r) = (A_S \exp(-1.55r) + 1438.72\exp(-3.11r))/r \quad (24)$$

where $V_S(r)$ is in MeV and r is in fm units. The attractive Yukawa term is defined with $A_{s=0} = -513.968\ MeV \cdot fm$ and $A_{s=1} = -626.885\ MeV \cdot fm$ for the two-nucleon interaction in spin singlet and triplet states respectively.

MT I-III potential has been chosen for two reasons. On one hand it is widely employed potential for which accurate benchmark calculations exist. On the other hand this potential, being a combination of the attractive and repulsive Yukawa terms, reflects well the structure of the realistic nucleon-nucleon interaction: it is strongly repulsive at the origin, however has narrow attractive well situated at $r \approx 1$ fm. Note that many numerical techniques fail for the potentials, like MT I-III, which contain a repulsive core.

In figure 1 we present our calculations for the NN ^{1}S$_0$ phaseshift at E_{cm} =1 MeV. Two calculation sequences have been performed by enforcing the $\overline{\psi}_l^{sc}(r)$ to vanish at the border of the numerical grid set at 50 fm and at 100 fm respectively, whereas complex scaling angle (θ) has been chosen to be 10° and 30°. The phaseshift is extracted from the $\overline{\psi}_l^{sc}(r)$ value at fixed distance r, according to eq. 5-6 (Coulomb free case) and eq. 9-10 (short range plus Coulomb interaction). Extracted phaseshift oscillates with r – this oscillatory behavior is due to the "premature" enforcement of $\overline{\psi}_l^{sc}(r)$ to vanish at the border of the grid $r_{\max}$. The phaseshifts extracted close to $r_{\max}$ are strongly affected by the cut-off and thus not reliable. The amplitude of the close-border oscillations is sizeably reduced by either increasing $r_{\max}$ or θ, i.e. by reducing the sharpness of the numerical cut-off. Extracted phaseshift from the calculation with $r_{\max} = 100$ fm and θ =30° is stable in a rather large window, which starts at $r \sim 5$ fm (right outside the interaction region) and extends up to $r \sim 70$ fm (beyond this value effect due to cut-off sets in). In the stability region extracted phaseshift value agrees well with the exact result (dotted line).

In figure 2 we compare NN ^{1}S$_0$ pahaseshift calculation for E_{cm}=1, 5 and 50 MeV by setting $r_{\max} = 100$ fm and θ =10°. One may see that by increasing energy effect of the cut-off reduces, sizeably improving stability of the extracted phaseshift. Inclusion of the repulsive Coulomb term does not have any effect on the quality of the method.

One may improve considerably accuracy of the phaseshift calculation by employing integral relation eq. (7), see tables I,II and figure 3. Phaseshift converges to constant value by either increasing cut-off radius $r_{\max}$ or complex rotation angle. Accuracy of five digits is easily reached. One should notice however that the use of very large values of θ should be avoided, due to the fact that the function $\overline{\psi}_l^{sc}(r)$ as well as complex scaled potential $V(re^{i\theta})$ might become very steep and rapidly oscillating. At higher energy the function $\overline{\psi}_l^{sc}(r)$ vanishes faster and thus one achieves convergence by employing smaller values of $r_{\max}$ and/or θ.

Our analysis has been extended to the three-body case. We consider nucleon-deuteron (N-d) $L = 0$ scattering in spin-doublet ($S = 1/2$) and quartet ($S = 3/2$) states. Calculations has been performed both below and above three-particle break-up threshold. Below the break-up threshold results are stable and independent of the scaling angle, similar to the two-body case. Phaseshifts might be accurately extracted both using differential and integral expressions.

Application of the differential relations for extracting scattering phaseshift and inelasticity above the break-up threshold is not so-obvious. It is difficult to find the stability domain. Therefore we employed integral expressions eqs. (23) and (19), obtained using Greens theorem, which once again proved their worth. We summarize obtained results in Table III and IV, respectively for n-d and p-d scattering above the break-up threshold. Very accurate results are obtained for both phaseshift and inelasticity parameter once complex scaling angle is chosen in the interval

TABLE I: Calculation of the scattering phaseshift using integral expressions at E_{cm} =1 MeV

	MT I-III				MT I-III+Coulomb			
r_{max} (fm)	5°	10°	30°	50°	5°	10°	30°	50°
10	44.420	49.486	55.790	56.676	33.999	36.390	41.528	43.805
25	34.704	44.211	62.654	63.743	24.772	34.910	50.693	50.698
50	56.812	61.083	63.482	63.512	39.895	46.546	50.487	50.491
100	66.502	63.822	63.512	63.512	55.463	50.811	50.491	50.491
150	62.497	63.485	63.512	63.512	49.317	50.474	50.491	50.491
exact	63.512				50.491			

TABLE II: Calculation of the scattering phaseshift using integral expressions at E_{cm} =50 MeV

r_{max} (fm)	MT I-III				MT I-III+Coulomb			
	3°	5°	10°	30°	3°	5°	10°	30°
10	19.400	19.719	19.923	19.605	19.795	20.245	20.610	20.313
25	20.788	20.135	20.027	20.032	21.530	20.864	20.755	20.760
50	20.014	20.026	20.027	20.027	20.734	20.754	20.755	20.755
100	20.027	20.027	20.027	20.027	20.755	20.755	20.755	20.755
exact	20.027				20.755			

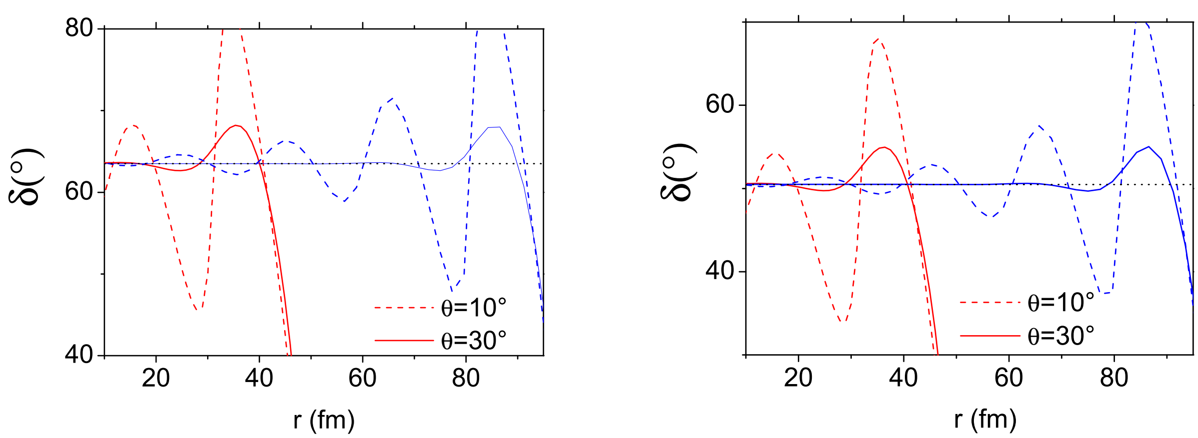

FIG. 1: (Color online) ^{1}S$_0$ NN phaseshift at E_{cm}=1 MeV extracted using relations eq. 5-6 and 9-10 respectively. Calculations performed with cut-off imposed at r_{max}=50 and 100 fm using complex rotation by the angle θ =10° (dashed lines) and θ =30° (solid line). Pure strong interaction result is presented in the left figure, calculations including repulsive Coulomb interaction for pp-pair are presented in the right figure.

$[4°, 12.5°]$ for incident neutron with energy E$_{lab}$=14.1 MeV and $[3°, 7.5°]$ at E$_{lab}$=42 MeV. Stability of the final result within at least three significant digits is assured, providing excellent agreement with the benchmark calculations of [23, 24]. Calculated integral gradually cease to converge on the finite domain for the calculations when higher complex scaling angles are chosen. This happens due to the fact that condition eq.(18), which set limit θ_{max}=14.2° and θ_{max}=8.9° for the calculations at E$_{lab}$=14.1 MeV and E$_{lab}$=42 MeV respectively, is violated.

In the table V we tabulate $^3S_1$ n-d break-up amplitude as a function of the break-up angle ϑ, which defines pair and spectator wave numbers via k=K cos(ϑ) and q= 2K sin(ϑ)/$\sqrt{3}$ respectively. Nice agreement is obtained with the benchmark calculation of [23]. Small discrepancy appears only for the ϑ values close to 90°, which defines configuration where one pair of particles after the break-up remains at rest. This is due to the slow convergence of the integral eq.(19) for $\vartheta \to 90°$ in y-direction, special procedure must be undertaken in this particular case to evaluate the part of the slowly convergent integral outside the resolution domain defined by y$_{max}$.

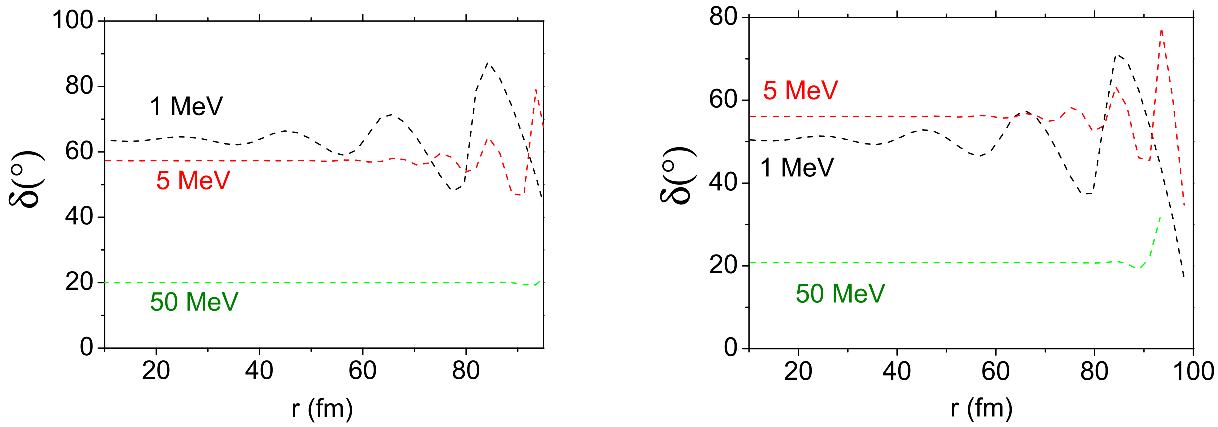

FIG. 2: (Color online) ${}^{1}S_0$ NN phaseshift calculation at E_{cm}=1, 5 and 50 MeV extracted using relations eq. 5-6 and 9-10 respectively. Calculations performed with cut-off imposed at r_{max}= 100 fm using complex rotation by the angle θ =10°. Pure strong interaction result is presented in the left figure, calculations including repulsive Coulomb interaction for pp-pair are presented in the right figure.

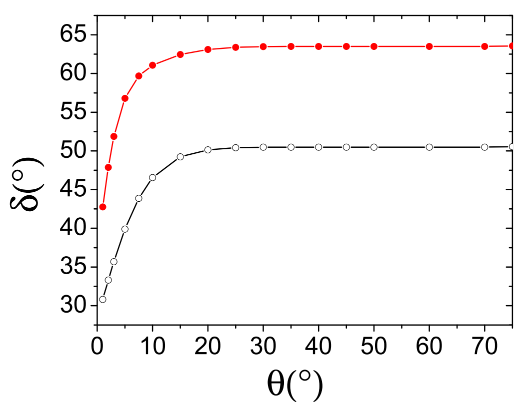

FIG. 3: (Color online) Dependence of the calculated NN ${}^{1}S_0$ phaseshift using integral expression as a function of the complex rotation angle. Grid was limited to r_{max}=100 fm. The upper curve correspond Coulomb-free case, the bottom one includes Coulomb.

V. CONCLUSION

In this work we have presented a method based on the complex scaling, which enables to solve few-nucleon scattering problem without explicit treatment of the boundary conditions using square-integrable functions. Validity of the method is demonstrated for two and three particle scattering, including the three-particle break-up case with repulsive Coulomb interaction. Three-digit accuracy maybe easily obtained using this method.

As is well known for two body case complex scaling angle is, in principle, only limited to 90°. On the contrary in order to solve three-body break-up problem the scaling angle should be restricted stronger from above, see eq.(18). I.e. for the scattering at high energy the scaling angle should be limited to very small values. Nevertheless this limitation does not spoil the method at high energies, since rapid vanishing of the outgoing wave after the scaling is ensured by the large wave number values.

TABLE III: Neutron-deuteron scattering phaseshift and inelasticity parameter as a function of the complex rotation angle θ compared with benchmark results of [23, 24]. Our calculations has been performed by setting y_{max}=100 fm.

	3°	4°	5°	6°	7.5°	10°	12.5°	Ref. [23, 24]
				nd doublet at E_{lab}=14.1 MeV				
Re(δ)	105.00	105.43	105.50	105.50	105.50	105.49	105.48	105.49
η	0.4559	0.4638	0.4653	0.4654	0.4653	0.4650	0.4649	0.4649
				nd doublet at E_{lab}=42 MeV				
Re(δ)	41.71	41.63	41.55	41.51	41.45	41.04		41.35
η	0.5017	0.5015	0.5014	0.5014	0.5015	0.5048		0.5022
				nd quartet at E_{lab}=14.1 MeV				
Re(δ)	68.47	68.90	68.97	68.97	68.97	68.97	68.97	68.95
η	0.9661	0.9762	0.9782	0.9784	0.9783	0.9782	0.9780	0.9782
				nd quartet at E_{lab}=42 MeV				
Re(δ)	37.83	37.80	37.77	37.77	37.74	38.06	-	37.71
η	0.9038	0.9034	0.9032	0.9030	0.9029	0.8980	-	0.9033

TABLE IV: Proton-deuteron scattering phaseshift and inelasticity parameter as a function of the complex rotation angle θ compared with benchmark values of [24]. Our calculations has been performed by setting y_{max}=150 fm.

	3°	4°	5°	6°	7.5°	10°	12.5°	Ref. [24]
				pd doublet at E_{lab}=14.1 MeV				
Re(δ)	108.46	108.43	108.43	108.43	108.43	108.43	108.42	108.41[3]
η	0.5003	0.4993	0.4990	0.4988	0.4986	0.4984	0.4981	0.4983[1]
				pd doublet at E_{lab}=42 MeV				
Re(δ)	43.98	43.92	43.87	43.82	43.78	44.83	-	43.68[2]
η	0.5066	0.5060	0.5056	0.5054	0.5052	0.5488	-	0.5056
				pd quartet at E_{lab}=14.1 MeV				
Re(δ)	72.70	72.65	72.65	72.64	72.64	72.63	72.62	72.60
η	0.9842	0.9827	0.9826	0.9826	0.9826	0.9828	0.9829	0.9795[1]
				pd quartet at E_{lab}=42 MeV				
Re(δ)	40.13	40.11	40.08	40.07	40.05	40.35	-	39.96[1]
η	0.9052	0.9044	0.9039	0.9036	0.9034	0.9026	-	0.9046

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TABLE V: Neutron-deuteron $^3S_1$ break-up amplitude calculated at E_{lab}=42 MeV as a function of the break-up angle ϑ.

	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
This work Re($^3S_1$)	1.49[-2]	8.84[-4]	-3.40[-2]	3.33[-2]	7.70[-2]	2.52[-1]	4.47[-1]	6.47[-1]	6.30[-1]	-1.62[-1]
This work Im($^3S_1$)	1.69[0]	1.74[0]	1.87[0]	1.92[0]	1.80[0]	1.68[0]	1.70[0]	1.96[0]	2.23[0]	3.17[0]
Ref. [23] Re($^3S_1$)	1.48[-2]	9.22[-4]	-3.21[-2]	3.09[-2]	7.70[-2]	2.52[-1]	4.51[-1]	6.53[-1]	6.93[-1]	-1.05[-1]
Ref. [23] Im($^3S_1$)	1.69[0]	1.74[0]	1.87[0]	1.92[0]	1.80[0]	1.67[0]	1.70[0]	1.95[0]	2.52[0]	3.06[0]

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[25] One must note that this condition is derived for a system of three particles with identical masses. For the case of three particles with arbitrary masses, one gets: $\tan\theta < \sqrt{\frac{|\epsilon_n| M m_i}{(E+|\epsilon_n|) m_j m_k}}$; where m_i is the mass of incoming particle, the binding energy $|\epsilon_n|$ correspond the weakest bound state of the particle pair (jk) open for scattering, $M = m_i + m_j + m_k$ is a total mass of the system.
[26] For the case of three particles with arbitrary masses, one gets: $\tan\theta < \sqrt{m_i M |\epsilon_m|}/(\sqrt{m_j m_k(|\epsilon_m| + E)} + \sqrt{(M-m_j)(M-m_k)(|\epsilon_n| + E)})$
[27] In principle, Faddeev equations maybe solved also for long-range scattering problem after the complex scaling, however only vanishing of the total wave function and not of the separate Faddeev amplitudes (ones work with) is assured in this case.